

Heal the world: Avoiding the cosmic doomsday in the holographic dark energy model

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The current observational data imply that the universe would end with a cosmic doomsday in the holographic dark energy model. However, unfortunately, the big-rip singularity will ruin the theoretical foundation of the holographic dark energy scenario. To rescue the holographic scenario of dark energy, we employ the braneworld cosmology and incorporate the extra-dimension effects into the holographic theory of dark energy. We find that such a mend could erase the big-rip singularity and leads to a de Sitter finale for the holographic cosmos. Therefore, in the holographic dark energy model, the extra-dimension recipe could heal the world.

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I. INTRODUCTION

Dark energy was found by the observations of type Ia supernovae in 1998 [1]. It is believed that the current cosmic acceleration is caused by dark energy. The basic characteristic of dark energy is that its equation of state parameter w (the definition of w is $w = p/\rho$, where ρ is the energy density and p is the pressure) has a negative value ($w < -1/3$). At present, the universe is under the domination of dark energy. The combined analysis of cosmological observations shows that the universe is spatially flat and consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation [2].

Although it can be affirmed that the ultimate fate of the universe is determined by the feature of dark energy, the nature of dark energy as well as its cosmological origin still remain enigmatic at present. The preferred candidate for dark energy is the famous Einstein's cosmological constant λ [3], however, it always suffers from the "fine-tuning" and "cosmic coincidence" puzzles. The fine-tuning problem asks why the vacuum energy density today is so small compared to typical particle scales. The vacuum energy density is of order 10^{-47}GeV^4 , which appears to require the introduction of a new mass scale 14 or so orders of magnitude smaller than the electroweak scale. The second difficulty, the cosmic coincidence problem, asks: Since the energy densities of vacuum energy and dark matter scale so differently during the expansion history of the universe, why are they nearly equal today? To get this coincidence, it appears that their ratio must be set to a specific, infinitesimal value in the very early universe. Lots of efforts have been made to try to resolve the cosmological-constant problem, all these efforts, however, turned out to be unsuccessful [4, 5].

On the other hand, at the phenomenological level, many dynamical dark energy models have been proposed to interpret the observational data (for reviews, see, e.g., Refs. [6]). In terms of the equation of state, these dynamical dark energy scenarios can be classified into the following three categories: quintessence, phantom, and quintom. For quintessence-like

dark energy, the equation of state parameter is always greater than -1 , namely $w \geq -1$; for phantom-like dark energy, the equation of state parameter is always less than -1 , namely $w \leq -1$; for quintom-like dark energy, the equation of state parameter crosses -1 during the evolution. In the scenario of phantom dark energy, it is remarkable that all the energy conditions in general relativity (including the weak energy condition) are violated. A ghastful prediction of this scenario is the "cosmic doomsday" [8]. Due to the equation of state less than -1 , the phantom component leads to a "big rip" singularity at a finite future time, at which all bound objects will be torn apart. For detailed discussions on the properties of future singularities of the universe, see Ref. [9].

Possibly, the cosmological constant (or the vacuum energy density) might also have some dynamical property. It is well known that the cosmological constant is actually closely related to an ultraviolet (UV) problem in the quantum field theory. A simple evaluation in quantum field theory leads to a discrepancy of 120 orders of magnitude between the theoretical result and the observational one [4]. Obviously, the key point is the gravity. In a real universe, the effects of gravity should be involved in this evaluation. So, actually, the cosmological constant (or dark energy) problem is in essence an issue of quantum gravity [7]. However, by far, we have no a complete theory of quantum gravity, so it seems that we have to consider the effects of gravity in some effective quantum field theory in which some fundamental principles of quantum gravity should be taken into account. It is commonly believed that the holographic principle [10] is just a fundamental principle of quantum gravity. Taking the holographic principle into account, dynamical vacuum energy is possible.

The holographic principle is expected to play an important role in dark energy research. When considering gravity, namely, in a quantum gravity system, the conventional local quantum field theory will break down due to the too many degrees of freedom that would cause the formation of black hole. So, there is a proposal saying that the holographic principle may put an energy bound on the vacuum energy density, $\rho_{vac} L^3 \leq M_{pl}^2 L$, where ρ_{vac} is the vacuum energy density and M_{pl} is the reduced Planck mass [11]. This bound says that the total energy in a spatial region with size L should not exceed the mass of a black hole with the same size. The largest size compatible with this bound is the infrared (IR) cutoff size

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of this effective field theory. Evidently, this bound implies a UV/IR duality. Therefore, the holographic principle may lead to a dark energy model that is actually based on the effective quantum field theory with a UV/IR duality. From this UV/IR correspondence, the UV problem of dark energy can be converted into an IR problem.

By phenomenologically introducing a dimensionless parameter c , one can saturate that bound and write the dark energy density as $\rho_{de} = 3c^2 M_{pl}^2 L^{-2}$. The parameter c is phenomenologically introduced to characterize all of the uncertainties of the theory. Now, the problem becomes how to choose an appropriate IR cutoff for the theory. A natural choice is the Hubble length of the universe, however, it has been proven that there is no cosmic acceleration for this choice [12]. Li proposed that, instead of the Hubble horizon, one can choose the event horizon of the universe as the IR cutoff of the theory [13]. This choice not only gives a reasonable value for dark energy density, but also gives rise to an acceleration solution for the cosmic expansion.

The parameter c in the holographic dark energy model plays a very important role in determining the final fate of the universe [14, 15]. In particular, when c is less than 1, the equation of state of holographic dark energy will evolve across the cosmological-constant boundary $w = -1$. Note that it will evolve from the region of $w > -1$ to that of $w < -1$, so the choice of $c < 1$ makes the holographic dark energy finally become a phantom energy that would lead to a cosmic doomsday (“big rip”) in the future. It should be pointed out that the holographic dark energy model has been strictly constrained by the current cosmological observations [16, 17, 18]. The joint analysis of the latest observational data, including type Ia supernovae (SN), cosmic microwave background (CMB), and baryon acoustic oscillation (BAO), shows that the parameter c is indeed less than 1 (at nearly 2σ level): $c = 0.818^{+0.113}_{-0.097} (1\sigma) {}^{+0.196}_{-0.154} (2\sigma)$ [18]. Thus, it seems that the big rip is inevitable in the holographic dark energy model. However, on the other hand, in the framework of holographic dark energy model, the big rip is actually not allowed, due to the reason that the Planck scale excursion of UV cutoff in the effective field theory is forbidden. So, the occurrence of the cosmic doomsday would canker the theoretical root of the holographic dark energy scenario.

To rescue the holographic dark energy model, we have to try to find out some working mechanism to erase the big-rip singularity in the phantom regime of the holographic dark energy scenario. In this paper, we will explore such a mechanism remedied by which the holographic theory of dark energy can be healed. We find that the extra dimension mechanism would provide us with a good cure for the illness of the holographic dark energy model. In what follows, we will incorporate the extra dimension effects into the holographic dark energy scenario, and we will see that such a mend works very well: the big-rip singularity would be eliminated successfully. Furthermore, we will find that the ultimate fate of the cosmos is an attractor where the steady state (de Sitter) finale occurs.

This paper is organized as follows: In Sec. II, we briefly review the holographic dark energy model and show that the

cosmic doomsday seems inevitable in this model according to the current cosmological observations. We also expatiate on the necessity of eliminating the big-rip singularity in the holographic model of dark energy. In Sec. III, we employ the braneworld mechanism to rescue the holographic dark energy model. We show that the big-rip singularity would be erased successfully by using the extra dimension mechanism, and the cosmic destiny would be a de Sitter phase. Furthermore, we give possible constraint on the holographic dark energy model from the extra dimension prescription. Finally, we give the conclusion in Sec. IV.

II. COSMIC DOOMSDAY IN HOLOGRAPHIC DARK ENERGY SCENARIO

The holographic dark energy model proposed by Li [13] is based on the future event horizon as an IR cutoff. The dark energy density is written as

$$\rho_{de} = 3c^2 M_{pl}^2 R_{eh}^{-2}, \quad (1)$$

where R_{eh} is the event horizon of the universe, which is defined as

$$R_{eh}(t) = a(t) \int_t^\infty \frac{dt'}{a(t')}. \quad (2)$$

From the definition of the event horizon (2), we can easily derive

$$\dot{R}_{eh} = H R_{eh} - 1. \quad (3)$$

So, taking derivative of Eq. (1) with respect to time t and using the energy conservation equation $\dot{\rho}_{de} + 3H(1+w)\rho_{de} = 0$, we can obtain the equation of state of holographic dark energy,

$$w = -\frac{1}{3} - \frac{2}{3c} \sqrt{\Omega_{de}}, \quad (4)$$

where

$$\Omega_{de} = \frac{\rho_{de}}{3M_{pl}^2 H^2} = \frac{c^2}{H^2 R_{eh}^2} \quad (5)$$

is the fractional density of holographic dark energy. For convenience, hereafter, we will use the units with $M_{pl} = 1$, but we will still explicitly write out M_{pl} at several places. To see the evolution dynamics of the holographic dark energy, we take derivative of Eq. (5) with respect to $\ln a$, and derive

$$\Omega'_{de} = 2\Omega_{de} \left(\epsilon - 1 + \frac{\sqrt{\Omega_{de}}}{c} \right), \quad (6)$$

where $\epsilon \equiv -\dot{H}/H^2 = -H'/H$, and a prime denotes the derivative with respect to $\ln a$. Using the Friedmann equation $3H^2 = \rho_m + \rho_{de}$ and the equation of state of dark energy (4), we have

$$\epsilon = \frac{3}{2}(1 + w_{de}\Omega_{de}) = \frac{3}{2} - \frac{\Omega_{de}}{2} - \frac{\Omega_{de}^{3/2}}{c}. \quad (7)$$

Hence, we obtain the equation of motion, a differential equation, for Ω_{de} ,

$$\Omega'_{de} = \Omega_{de}(1 - \Omega_{de}) \left(1 + \frac{2}{c} \sqrt{\Omega_{de}} \right). \quad (8)$$

A. Cosmic doomsday

The parameter c plays a significant role for the cosmological evolution of the holographic dark energy. When $c \geq 1$, the equation of state of dark energy will evolve in the region of $-1 \leq w \leq -1/3$. In particular, if $c = 1$ is chosen, the behavior of the holographic dark energy will be more and more like a cosmological constant with the expansion of the universe, such that ultimately the universe will enter the de Sitter phase in the distant future. When $c < 1$, the holographic dark energy will exhibit a quintomlike evolution behavior (for “quintom” dark energy, see, e.g., Refs. [19] and references therein); i.e., the equation of state of holographic dark energy will evolve across the cosmological-constant boundary $w = -1$ (actually, it evolves from the region with $w > -1$ to that with $w < -1$). That is to say, the choice of $c < 1$ makes the holographic dark energy behave as a quintom energy that would lead to a cosmic doomsday (“big rip”) in the future. Thus, as discussed above, the value of c determines the destiny of the universe in the holographic dark energy model.

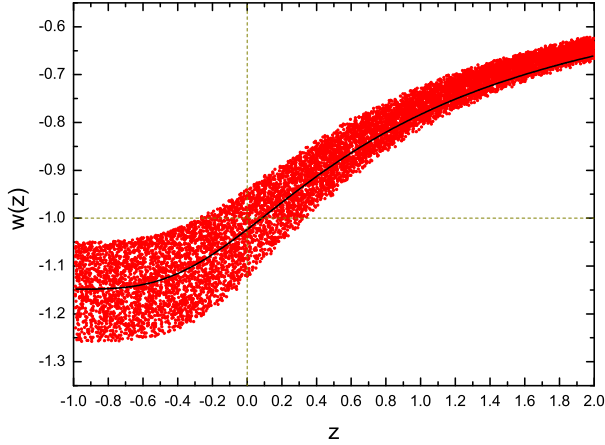


FIG. 1: The evolution of the equation of state of holographic dark energy $w(z)$ with 1σ uncertainty. In this figure, the central black solid line represents the best fit, and the red dotted area around the best fit covers the range of 1σ errors. The errors quoted here are calculated using a Monte Carlo method where random points are chosen in the 1σ region of the parameter space: $c = 0.818^{+0.113}_{-0.097}$ and $\Omega_{m0} = 0.277^{+0.022}_{-0.021}$. From this figure, one can clearly see that our universe is striding into the phantom regime of the holographic dark energy scenario.

The holographic dark energy model has been strictly constrained by the current cosmological observations. The joint analysis of the latest observational data including the Constitution sample of 397 SN, the shift parameter of the CMB given by the five-year Wilkinson Microwave Anisotropy Probe

(WMAP5) observations, and the BAO measurement from the Sloan Digital Sky Survey (SDSS), shows that the parameter c is indeed less than 1 (at nearly 2σ confidence level): $c = 0.818^{+0.113}_{-0.097} (1\sigma) {}^{+0.196}_{-0.154} (2\sigma)$ [18]. Using the fitting result of Ref. [18], we generate in Fig. 1 the evolution of the equation of state of holographic dark energy $w(z)$ with 1σ uncertainty. In this figure, the central black solid line represents the best fit, and the red dotted area around the best fit covers the range of 1σ errors. The errors quoted in Fig. 1 are calculated using a Monte Carlo method where random points are chosen in the 1σ region of the parameter space shown in Fig. 1 of Ref. [18]. From Fig. 1, one can clearly see that our universe is striding into the phantom regime in the holographic dark energy scenario. So, according to the current observational data, it seems that the cosmic doomsday is inevitable in this scenario. However, after detailedly investigating the theoretical foundation of the holographic dark energy, one might ask such a question: Is the cosmic doomsday really allowed in the holographic scenario?

B. The physical motivation: Why must eliminate the big-rip singularity in the holographic dark energy model?

First of all, it should be necessary to review the theoretical foundation of the holographic dark energy model. In fact, the holographic dark energy model is based on the effective quantum field theory with some UV/IR relation [11, 13]. Obviously, this is not the case in particle physics. In general, for particle physics, one only needs an effective field theory with a UV cutoff. It is usually assumed that the properties of elementary particles can be accurately described by an effective field theory with a UV cutoff less than the Planck mass M_{pl} , provided that all momenta and field strengths are small compared with this cutoff to the appropriate power. The standard model of particle physics provides a good example for this. However, when gravity is considered in the system, especially when black holes are involved, the underlying theory of nature is suggested to be not a local quantum field theory. Under the situation that a complete theory of quantum gravity is not available, a good way of accurately describing the world is to try to use the effective quantum field theory in which the effects of gravity are adequately taken into account and the range of validity for the local effective field theory is determined. To accomplish this, some relationship between UV and IR cutoffs should be imposed. Of course, the proposed IR bound should not conflict with any current experimental success of quantum field theory.

Local quantum field theory could not be a good effective low energy description of any system containing a black hole, and should not attempt to describe particle states whose volume is smaller than their corresponding Schwarzschild radius. Nevertheless, in an effective quantum field theory, for any UV cutoff Λ , there is a sufficiently large volume for which the vastly overcounted degrees of freedom of the effective field theory would lead to the formation of a black hole spoiling the effective local quantum field theory. To avoid this difficulty, Cohen et al. [11] propose a constraint on the IR cutoff $1/L$

which excludes all states that lie within their Schwarzschild radius, namely, $L^3 \Lambda^4 \lesssim LM_{pl}^2$. This bound implies a UV/IR duality since the IR cutoff scales like Λ^{-2} . So, in fact, they propose an effective local quantum field theory in which the UV and IR cutoffs are not independent and only those states that can be described by conventional quantum field theory are considered. The holographic dark energy model is nothing but such an effective quantum field theory with the IR cutoff length scale chosen to be the size of the event horizon of the universe [13]. In this model, the UV cutoff Λ runs with the cosmological evolution since the event horizon of the universe as the IR cutoff scale varies. Therefore, the holographic dark energy is actually a dynamical vacuum energy [15].

Let us have a look at the late-time evolution of the holographic cosmos. For the future event horizon, from Eq. (5), we have $R_{eh} = c/(H\sqrt{\Omega_{de}})$. Hence, in the far future where the dark energy totally dominates and other energy components are diluted away, the event horizon behaves as $R_{eh} = cH^{-1}$. Using Eqs. (3) and (7), we can further obtain $\dot{R}_{eh} = c - 1$. Now, one can clearly see that the parameter c plays a crucial role in determining the ultimate fate of the universe: when $c = 1$, we have $\dot{R}_{eh} = 0$ that corresponds to a de Sitter space-time; when $c > 1$, \dot{R}_{eh} is a positive constant indicating that R_{eh} increases with a constant change rate; when $c < 1$, we see that \dot{R}_{eh} is a negative constant implying that R_{eh} diminishes with a constant change rate. Thus, $c < 1$ will lead to a big-rip singularity at which R_{eh} reaches zero and other quantities such as the cosmological scale factor a , the dark energy density ρ_{de} and the Hubble expansion rate H approach infinity. However, the appearance of the big rip is definitely beyond the scope of the effective quantum field theory described above. Prior to the big rip, the UV cutoff Λ will first exceed the Planck mass M_{pl} , and then even the IR cutoff scale R_{eh} will become smaller than the Planck length l_{pl} . Obviously, such a super-Planck phenomenon will break down the theoretical foundation of the holographic dark energy model. So, it should be confessed that the holographic dark energy model undoubtedly has some congenital flaw in the phantom regime in some sense. In view of the successes of the holographic dark energy model in explaining the theoretical puzzles of dark energy [13, 20] and fitting the observational data [16, 17, 18], one should remedy this model in the high energy regime to make the holographic theory of dark energy more consistent and successful. The key for this is to find out a working mechanism to eliminate the big-rip singularity in this model.

C. How to erase the big-rip singularity?

It is anticipated that some unknown high-energy physical effects, especially those from the quantum gravity, might play an important role shortly before the would-be big rip. Hence, conventionally, we expect that when considering the effects of some unknown physics at high energies, the big rip would be erased in the holographic dark energy model. Now a question naturally arises asking what kind of effects appears likely to be the right mechanism of amending the behavior of holographic dark energy in high energy regime. In this subsection, we shall

discuss this issue.

It is the value of the parameter c that determines whether the big rip occurs: the appearance of the big rip is closely related to the fact of $c < 1$. So, to rescue the holographic dark energy model, one may expect that at ultra high energies some quantum gravity effects would lead to some correction to the parameter c , which makes the parameter c effectively change to be equal to or greater than one. That is to say, we must impose some mechanism that leads to the high energy corrections to c to realize that $c_{eff} \geq 1$ at high energies, where the effective parameter c_{eff} is presumed to be of the form

$$c_{eff}(t) = c + \text{correction from high energies.} \quad (9)$$

For the case of $c < 1$, when the holographic dark energy is in the low energy regime, it is obvious that $c_{eff} \rightarrow c$; when the holographic phantom energy enters the high energy regime, the quantum gravity is expected to begin to impact and consequently c gets a corresponding correction like Eq. (9). Of course, when $c \geq 1$, there is no high energy regime for dark energy, so we always have $c_{eff} = c$ for these cases.

In order to erase the big rip, $c_{eff} \geq 1$ prior to the would-be big rip must be satisfied. However, the most natural anticipation is presumed to be $c_{eff} \rightarrow 1$ for which the steady state (de Sitter) finale will emerge. Moreover, such a finale is expected to be an attractor solution. Can such a dramatic mechanism really be found out and performed in the holographic dark energy scenario? In the next section, we shall accomplish this picture by employing the extra-dimension mechanism (braneworld scenario).

III. HOLOGRAPHIC DARK ENERGY IN BRANEWORLD COSMOLOGY

In the present section, we shall discuss the scenario of holographic dark energy in Randall-Sundrum (RS) braneworld. We will find that the cosmic doomsday could be avoided successfully and the de Sitter finale would emerge as an attractor.

A. Why braneworld?

In the holographic dark energy model, the effects of gravity have been adequately considered by using the holographic principle to impose a UV/IR relationship in an effective local quantum field theory, however, another important aspect of the spacetime, extra dimensions, is absent. When the energy scale of dark energy is low enough, the effects from extra dimensions are absolutely negligible; however, when the phantom energy density becomes enormously high, the extra-dimension effects are expected to play a significant role. Accordingly, in order to make the holographic dark energy model more complete, the extra dimensions should also be considered. In addition, another reason for supporting the involvement of the effects of extra dimensions in the holographic dark energy model comes from the fact that the braneworld scenario might provide us with some positive correction to c as discussed in Sec. II C. So, it is quite interesting to study how

the physics of extra dimensions may affect the behavior of the holographic dark energy in phantom regime. In the following we will focus on a simple braneworld case.

Considering the case with one extra dimension compactified on a circle, the effective four-dimensional Friedmann equation is [21, 22]

$$3H^2 = \rho \left(1 + \frac{\rho}{\rho_c} \right), \quad (10)$$

where $\rho_c = 2\sigma$, with σ the brane tension,

$$\sigma = \frac{6(8\pi)^2 M_*^6}{M_{pl}^2}, \quad (11)$$

where M_* is the true gravity scale of the five-dimensional theory, and in this expression we explicitly write out M_{pl} . In general, the most natural energy scale of the brane tension is of the order of the Planck mass, but the problem can be generally treated for any value of $\sigma > \text{TeV}^4$.

It can be explicitly seen from the modified Friedmann equation (10) that the extra-dimension physics could contribute some positive correction to the effective parameter c_{eff} , namely,

$$c_{eff}(t) = c \sqrt{1 + 3c^2 \rho_c^{-1} R_{eh}^{-2}(t)}. \quad (12)$$

Therefore, for making the holographic dark energy model more complete and consistent, it is quite natural to have recourse to the extra dimension scenario.

B. Cosmological evolution at late times

In this subsection, we will discuss the late-time evolution of the holographic dark energy in a braneworld, and derive the evolution equation.

At the late times, the universe is totally dominated by the holographic phantom energy, so in Eq. (10) we have $\rho = \rho_{de}$. Note that for any cases Eqs. (3)–(6) are always satisfied since they come from the definitions of holographic dark energy (1) and event horizon (2). In order to obtain the equation of motion of dark energy, we need to calculate ϵ in Eq. (6).

Taking the derivative of Eq. (10) with respect to time t and using the energy conservation equation $\dot{\rho}_{de} = -3H(1+w)\rho_{de}$, we can get

$$\epsilon = \frac{3}{2} \Omega_{de} (1+w) \left(1 + 2 \frac{\rho_{de}}{\rho_c} \right), \quad (13)$$

where the fractional density of dark energy is still defined as $\Omega_{de} = \rho_{de}/(3H^2)$, hence we have

$$\frac{\rho_{de}}{\rho_c} = \frac{1}{\Omega_{de}} - 1. \quad (14)$$

Substituting Eqs. (4) and (14) into Eq. (13), we thus get

$$\epsilon = (2 - \Omega_{de}) \left(1 - \frac{1}{c} \sqrt{\Omega_{de}} \right). \quad (15)$$

Substituting Eq. (15) into Eq. (6), we eventually obtain the equation of motion for holographic phantom dark energy ($c < 1$) in a braneworld,

$$\Omega'_{de} = 2\Omega_{de}(1 - \Omega_{de}) \left(1 - \frac{1}{c} \sqrt{\Omega_{de}} \right). \quad (16)$$

This differential equation governs the dynamical evolution of the holographic dark energy when the effects of extra dimension begin to play an important role. Therefore, the whole picture of the holographic dark energy model is actually jointed by the two segments: before the extra dimension effects work, the dynamics of the holographic dark energy is dictated by Eq. (8); after the extra dimension effects emerge, the holographic dark energy model will be governed by Eq. (16). For the former stage, the universe will finally be totally dominated by the phantom energy, so at late times of this stage we have $\Omega_{de} = 1$; this is a stable attractor of Eq. (8). When the evolution of the phantom energy intrudes into the ultra high energy regime and the extra dimension mechanism begins to operate, we find that Ω_{de} begins to decrease with the expansion of the universe though the phantom energy density ρ_{de} always increases. From Eq. (14), we see that assuredly Ω_{de} decreases with the increase of ρ_{de} . Furthermore, from Eq. (16), one can find that Ω_{de} will decrease from 1 to a stable value c^2 that corresponds to the final state of the universe, so $\Omega_{de} = c^2$ should be the late-time attractor for the holographic phantom energy in the RS braneworld.

C. Finale of the universe: De Sitter spacetime attractor

We will show that the finale of the universe in this scenario is a de Sitter (steady state) spacetime.

First, let us prove that $\Omega_{de} = c^2$ is a stable late-time attractor solution to Eq. (16). Considering a perturbation to this solution, we find that the solution will be recovered soon: when $\Omega_{de} < c^2$, owing to that $1 - \Omega_{de} > 0$ and $1 - \frac{1}{c} \sqrt{\Omega_{de}} > 0$, we have $\Omega'_{de} > 0$ indicating that Ω_{de} will increase until it reaches c^2 ; when $c^2 < \Omega_{de} < 1$, since $1 - \Omega_{de} > 0$ and $1 - \frac{1}{c} \sqrt{\Omega_{de}} < 0$, we have $\Omega'_{de} < 0$ implying that Ω_{de} will keep on decreasing until it touches c^2 . So, out of question, $\Omega_{de} = c^2$ is a stable late-time attractor solution.

In this stage, the Hubble expansion rate will increase until it becomes a constant. For convenience, we define

$$\tilde{h}^2 \equiv \frac{H^2}{\rho_c} = \frac{1 - \Omega_{de}}{3\Omega_{de}^2}, \quad (17)$$

and we find that the maximum of \tilde{h} is

$$\tilde{h}_{max} = \frac{\sqrt{1 - c^2}}{\sqrt{3}c^2}, \quad (18)$$

which corresponds to the late-time attractor solution $\Omega_{de} = c^2$. Therefore, we can draw the conclusion that the finale of the universe in this scenario is a de Sitter spacetime. At the finale, using Eqs. (9) and (14), one can check that $c_{eff} = 1$. Also,

from Eq. (9), one derives the minimum of the size of the event horizon,

$$R_{eh}^{min} = \frac{\sqrt{3}c^2}{\sqrt{(1-c^2)\rho_c}}. \quad (19)$$

By far, in this modified holographic dark energy model (with $c < 1$), it is of interest to find that the universe begins with an inflation and also ends with another inflation.

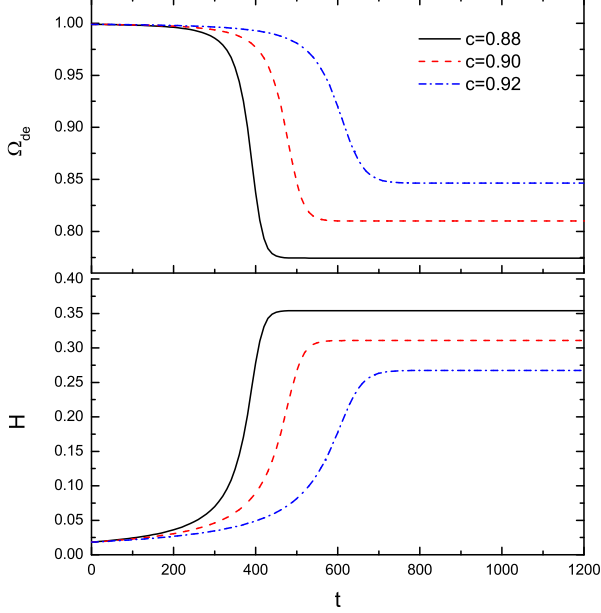


FIG. 2: The late-time evolution of Ω_{de} and H in the scenario of holographic dark energy in a RS braneworld. Here we take three cases as example, namely, $c = 0.88, 0.90$ and 0.92 . From this figure, we can explicitly see that the finale of the universe in this scenario is a steady-state spacetime. Note that in this plot the Hubble expansion rate H is in units of $(\sqrt{\rho_c}/M_{pl})$ and the cosmic time t is in units of $(M_{pl}/\sqrt{\rho_c})$.

As an example, we plot the late-time evolution curves of Ω_{de} and H according to the solution of the differential equation (16), as shown in Fig. 2. The initial condition of the calculation of Eq. (16) is taken as $\Omega_{de} = 1^-$ when $t = 0$. Here we take three cases as example, namely, $c = 0.88, 0.90$ and 0.92 . From this figure, we can explicitly see that the finale of the universe in this scenario is a steady-state spacetime. Note that in this plot the Hubble expansion rate H is in units of $(\sqrt{\rho_c}/M_{pl})$ and the cosmic time t is in units of $(M_{pl}/\sqrt{\rho_c})$.

So far, we have constructed a complete model of holographic dark energy in which the evolution of the universe is divided into two stages: In the low-energy regime, the dynamical evolution of the universe is governed by Eq. (8); in the high-energy regime, the dynamical evolution of the universe is dictated by Eq. (16). Therefore, the ultimate fate of the universe in the case of $c < 1$ should be a steady-state spacetime, instead of a cosmic doomsday. The consideration of an extra-dimension mechanism provides a successful solution to the theoretical puzzle of the holographic dark energy model

in the presence of a cosmic doomsday. Moreover, in quantum gravity theories such as the string/M theory, the spacetime is commonly believed to be fundamentally higher dimensional, so the involvement of the extra dimensions makes the holographic dark energy model more complete. In addition, it should be noted that the high-energy regime only exists in the case of $c < 1$, and in the cases of $c \geq 1$ there is no high-energy regime so that the extra-dimension effects will be absent in those cases.

It is conceivable that the parameter c of the holographic dark energy may be constrained by the braneworld cosmology. The IR cutoff length scale R_{eh} in the effective quantum field theory describing the holographic dark energy gets a minimum when the universe goes into the steady-state finale, as given by Eq. (19). If we impose a plausible additional requirement on R_{eh}^{min} that the IR cutoff length scale should be greater than the compactification radius of the extra dimensional circle, namely, $R_{eh}^{min} \gtrsim l$, where $l = 2\sqrt{3}\rho_c^{-1/2}$ is the anti de Sitter length scale, we will derive the constraint $c \gtrsim 2(\sqrt{2} - 1)$ (namely, $c \gtrsim 0.91$) that is consistent with the fitting result of the observational data [18]. It should also be noted that this constraint result should not be taken so seriously because actually the additional requirement on IR cutoff does not seem to be necessary.

One may also consider another possible braneworld scenario in which the effective four-dimensional Friedmann equation is $3H^2 = \rho(1 - \rho/\rho_c)$, where the negative sign arises from a second timelike dimension [23]. Note that this modified Friedmann equation can also arise from the loop quantum cosmology. Such a modified Friedmann equation with matter and phantom energy components can lead to a cyclic universe scenario in which the universe oscillates through a series of expansions and contractions [24]. Can this braneworld scenario help to eliminate the big-rip singularity in the holographic dark energy model? In fact, the high energy correction to c in this case is negative, namely, $c_{eff} = c\sqrt{1 - 3c^2\rho_c^{-1}R_{eh}^{-2}}$, so the cosmic doomsday finale could not be replaced by a steady state one in this case. One may further conceive that perhaps the holographic dark energy model combined with this scenario would replace the big-rip singularity with a turnaround. However, this is also impossible since a cyclic universe has no a future event horizon such that the definition of holographic dark energy breaks down in this scenario. In Ref. [25], such a scenario is investigated, but to evade this difficulty the future event horizon is redefined. So, it should be confessed that the scenario discussed in Ref. [25] is not realistic but only a toy model. The steady state future of the holographic Ricci dark energy in RS braneworld has been discussed in Ref. [26]. Other scenarios describing the holographic dark energy in braneworld can be found in, e.g., Refs. [27].

In addition, to avoid the big rip, it also seems quite natural to consider some interaction between holographic dark energy and matter [20, 28]. However, from the definition of the holographic dark energy (1), one can see that it is closely related to the future event horizon R_{eh} that is a global concept of spacetime. So, unlike the interaction between scalar-field dark energy and dark matter [29], it is rather difficult to imag-

ine the local interaction between holographic dark energy and matter. Therefore, we feel that the extra dimension recipe is better than the interaction one for solving the big rip crisis in the holographic dark energy model.

D. The whole story: From past to future

Finally, let us derive the evolution equation describing the whole expansion history from the past to the far future for the holographic dark energy model in a RS braneworld.

Consider, now, a general case that the universe contains dust matter (dark matter plus baryons) and holographic dark energy, namely, $\rho = \rho_m + \rho_{de}$, where $\rho_m = \rho_{m0}a^{-3}$ and $\rho_{de} = \rho_{de0}f(a)$ with $f(a) = \rho_{de}(a)/\rho_{de0}$. Note that here the subscript “0” marks the quantities corresponding to today, and for the scale factor of the universe a we have let $a_0 = 1$. From the modified Friedmann equation (10), one can easily derive

$$E(a)^2 = \left(\Omega_{m0}a^{-3} + \Omega_{de0}f(a) \right) \left[1 + \beta \left(\Omega_{m0}a^{-3} + \Omega_{de0}f(a) \right) \right], \quad (20)$$

where $E \equiv H/H_0$, $\Omega_{m0} = \rho_{m0}/(3H_0^2)$, $\Omega_{de0} = \rho_{de0}/(3H_0^2)$, and $\beta = 3H_0^2/\rho_c$. The dimensionless parameter β characterizes the ratio of the present-day density ρ_0 to the critical density of the braneworld ρ_c . From the fact that $\rho_0 \simeq 4 \times 10^{-47} \text{ GeV}^4$ and ρ_c takes some value between 10^{12} and 10^{72} GeV^4 , we estimate that the value of β lies between 10^{-119} and 10^{-59} . For a given Ω_{m0} , one can get $\Omega_{de0} = (\sqrt{1 + 4\beta} - 1)/2\beta - \Omega_{m0}$ from Eq. (20) by using the conditions $E_0 = E(t_0) = 1$ and $f_0 = f(t_0) = 1$. Also, by definition, the fractional densities of matter and dark energy can be expressed as

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{\Omega_{m0}a^{-3}}{E^2}, \quad \Omega_{de} = \frac{\rho_{de}}{3H^2} = \frac{\Omega_{de0}f(a)}{E^2}. \quad (21)$$

From Eq. (10) one can calculate $\epsilon = -\dot{H}/H^2$ and obtains

$$\epsilon = \frac{3}{2}[(1+w)\Omega_{de} + \Omega_m] \left(1 + 2\frac{\rho}{\rho_c} \right), \quad (22)$$

and also from Eq. (10) one derives

$$\frac{\rho}{\rho_c} = \frac{1}{\Omega_{de} + \Omega_m} - 1. \quad (23)$$

Consequently, in combination with Eq. (4) one obtains

$$\epsilon = \left(\frac{3}{2}\Omega_m + \Omega_{de} - \frac{1}{c}\Omega_{de}^{3/2} \right) \left(\frac{2}{\Omega_{de} + \Omega_m} - 1 \right). \quad (24)$$

To get the evolutionary behavior of holographic dark energy $f(a)$, one only needs to substitute Eq. (24) into Eq. (6) and solve the derived differential equation with the initial condition $f(t_0) = 1$. So far, we can describe the whole story of the holographic dark energy in a RS braneworld by using Eqs. (6), (20), (21) and (24).

As a schematic example, we take the case $c = 0.8$, $\Omega_{m0} = 0.27$ and $\beta = 10^{-10}$, and plot the expansion history of the universe, namely, $E(\ln a)$, in Fig. 3. Note that in this example

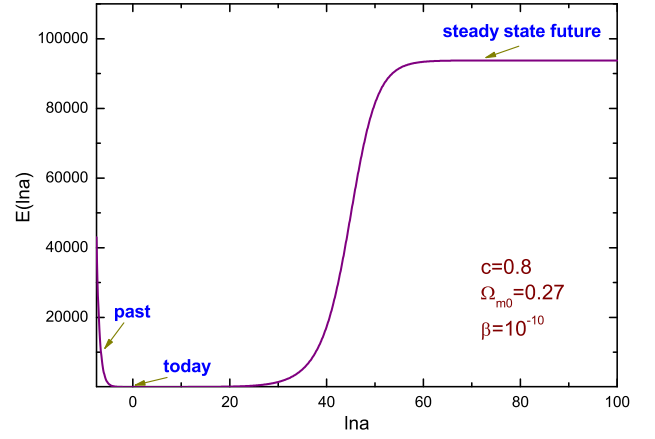


FIG. 3: The expansion history from the past to the far future for the holographic dark energy model in a RS braneworld. As a schematic example, here we take the case $c = 0.8$, $\Omega_{m0} = 0.27$ and $\beta = 10^{-10}$. Note that in this example the values of c and Ω_{m0} are realistic but the value of β is evidently unrealistic because it is much bigger than a reasonable value that is in the range of $10^{-119} - 10^{-59}$. This is only for the display effect of the plot.

the values of c and Ω_{m0} are realistic but the value of β is evidently unrealistic because it is much bigger than a reasonable value that is in the range of $10^{-119} - 10^{-59}$. This is only for the display effect of the plot. We remind the reader to notice the evolutionary trend of the universe shown in Fig. 3 but forget the unreality as a schematic example. The steady state (de Sitter) future can be explicitly identified in this figure. Also, it is shown in this figure that the past and the future of the expansion history in this scenario can be seamless linked.

IV. CONCLUSION

The holographic principle plays a very significant role in studying the dark energy problem. With the consideration of the holographic principle, the holographic dark energy model is proposed by constructing an effective quantum field theory with a UV/IR duality. Thanks to the UV/IR correspondence, the UV problem of dark energy can be converted into an IR problem. In the holographic dark energy model, the IR cutoff length scale is chosen as the size of the event horizon of the universe.

According to the observational data, it seems inevitable that the cosmic doomsday would be the ultimate fate of the universe in the holographic dark energy model. However, unfortunately, the big-rip singularity will undoubtedly ruin the theoretical foundation of the holographic dark energy scenario that is based upon an effective quantum field theory. To rescue the holographic dark energy model, we employ the braneworld cosmology and incorporate the extra-dimension effects into this model. The motivation of considering the extra dimensions consists of two aspects: (a) The spacetime is commonly believed to be fundamentally higher dimensional in quantum gravity theories such as string/M theory, so the

involvement of the extra dimensions would make the holographic dark energy model more complete. (b) With the help of the extra dimension mechanism, the big-rip singularity in the holographic dark energy model could be erased successfully.

In this paper, we have investigated the cosmological evolution of the holographic dark energy in the braneworld cosmology. It is of interest to find that for the far future evolution of the holographic dark energy in a RS braneworld, there is a late-time attractor solution where the steady state (de Sitter) finale occurs, instead of the big rip. Therefore, in the holo-

graphic dark energy model, the extra-dimension recipe could heal the world.

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